

Data Analysis

- (1) Page 663 – 664 #16 – 34 Column, 45 – 54 Column (Skip part B), and #57 (A S/S)
- (2) Page 663 – 664 #17 – 32 Column, 46 – 56 Column (Skip part B), and #60 (A S/S)
- (3) Page 670 – 671 #25 – 49 Column, 60 – 67 Column (Skip part B), and #70, 71 (G S/S)
- (4) Page 670 – 671 #26 – 50 Column, 61 – 69 Column (Skip part B), and #72, 73 (G S/S)
- (5) Collecting Data and Drawing Conclusions - Worksheet
- (6) Collecting Data and Drawing Conclusions - Worksheet
- (7) Line of Best Fit - Worksheet
- (8) Summary Statistics - Worksheet
- (9) Summary Statistics - Worksheet
- (10) Reading Graphs - Worksheet
- (11) Page 705 - 706 #15 - 35 Column and #55, 56 and 57 (FCP and P)
- (12) Page 712 - 713 #18 - 30 Column, #47 – 50 (Combinations)
- (13) Permutations and Combinations - Worksheet
- (14) Probability and Odds - Worksheet
- (15) Page 727 - 728 #16 - 39 Column, #42 and 43 (Probability of Compound Events)
- (16) Page 727 - 728 #18 - 40 Column, #44 – 46 (Probability of Compound Events)
- (17) Page 734 #1 – 11 all (Probability of Independent and Dependent Events)
- (18) Page 734 – 735 #12 – 24 all (Probability of Independent and Dependent Events)
- (19) Two Way Tables and Probability - Worksheet
- (20) Review packet for test tomorrow

Sequences and Series General Information

You can think of a _____ as a function whose domain is a set of consecutive integers. If a domain is not specified, it is understood that the domain starts with 1.

Finite sequences are ones that end.

Infinite sequences continue without stopping.

When the terms of a sequence are added, the resulting expression is a _____. A series can either be infinite or finite.

You can use _____ (aka sigma notation) to write a series

11.2 Arithmetic Sequences and Series (stress applications) (1/2)

In an _____, the difference between consecutive terms is constant. The constant difference is called the _____ and is denoted by d .

(E1.) Decide whether each set is arithmetic.

(a) -3, 1, 5, 9, 13, ...

(b) 2, 5, 10, 17, 26,...

(P1.) Decide whether each set is arithmetic.

(a) -10, -6, -2, 0, 2, 6, 10,...

(b) 5, 11, 17, 23, 29,...

The n th term of an arithmetic sequence with first term a_1 and common difference d is given by:

$$a_n = a_1 + (n - 1)d$$

Key:

a_1 - 1st term in the series or sequence

a_n - n th term in the series or sequence

n - location of a term in a series or sequence

d - common difference

r - common ratio

S_n - sum of the 1st to the n th term of a series

Σ - summation notation

*****To write a rule for the n th term in an arithmetic sequence, you must find a_1 and d .*****

(E2.) (a) Write a rule for the n th term of the sequence 50, 44, 38, 32,...

(b) Then find a_{20}

(P2.) (a) Write a rule for the n th term of the sequence 32, 47, 62, 77,...

(b) Then find a_{12}

(E3.) One term of an arithmetic sequence is $a_{13} = 30$. The common difference is $d = \frac{3}{2}$. Write a rule for the n th term.

(P3.) One term of an arithmetic sequence is $a_8 = 50$. The common difference is $d = .25$. Write a rule for the n th term.

The expression formed by adding the terms of an arithmetic sequence is called an arithmetic series. The sum of the first n terms of an arithmetic sequence is denoted by S_n .

The Sum of a Finite Arithmetic Series

The sum of the first n terms of an arithmetic series is: $S_n = n\left(\frac{a_1 + a_n}{2}\right)$

In words, S_n is the mean of the first n th terms, multiplied by the number of terms.

(E4.) Consider the arithmetic series $4 + 7 + 10 + 13 + 16 + 19 + \dots$ Find the sum of the first 30 terms.

(P4.) Consider the arithmetic series $20 + 18 + 16 + 14 + \dots$ Find the sum of the first 25 terms.

(E5.) The first row of a concert hall has 25 seats, and each row after the first one has one more seat than the row before it. There are 32 rows of seats. Write a rule for the number of seats in the n th row

(P5.) A construction company is laying a natural gas pipeline. Several sections of pipe have been laid in a pile at the construction site. There are 12 sections of pipe in the bottom row of the pile. Each row has

one less pipe than the row below it. There are 8 rows of pipe. Write a rule for the number of pipe sections in the n th row.

(E6.) The first row of a concert hall has 25 seats, and each row after the first one has one more seat than the row before it. There are 32 rows of seats. What is the total number of seats in the concert hall?

(P6.) A construction company is laying a natural gas pipeline. Several sections of pipe have been laid in a pile at the construction site. There are 12 sections of pipe in the bottom row of the pile. Each row has one less pipe than the row below it. There are 8 rows of pipe. What is the total number of pipe sections in the pile?

(E7.) Find the sum of the series

$$\sum_{i=1}^{10} (2 + i)$$

(P7.) Find the sum of the series

$$\sum_{i=1}^{15} 3 - i$$

In a _____, the ratio of any term to the previous term is constant. This constant ratio is called the common ratio and is denoted by r .

(E1.) Decide whether each sequence is geometric.

(a) 1, 2, 6, 24, 120,...

(b) 81, 27, 9, 3, 1,...

(P1.) Decide whether each sequence is geometric.

(a) 4, -8, 16, -32,...

(b) 3, 9, -27, -81, -243,...

Rule for a Geometric Sequence

The n th term of a geometric sequence with the first term a_1 and common ratio, r , is given by:

$$a_n = a_1(r)^{(n-1)}$$

(E2.) (a) Write a rule for the n th term of the sequence -8, -12, -18, -27,...

(b) Find a_8

(P2.) (a) Write a rule for the n th term of the sequence 5, 2, 0.8, 0.32.

(b) Find a_8

(E3.) One term of a geometric sequence is $a_3 = 5$. The common ratio is $r = 2$. Write a rule for the n th term.

(P3.) One term of a geometric sequence is $a_4 = 3$. The common ratio is $r = 3$. Write a rule for the n th term.

The expression formed by adding the terms of a geometric sequence is called a _____. As with an arithmetic series, the sum of the first n terms of a geometric series is denoted by S_n .

The Sum of a Finite Geometric Series with Common Ratio $r \neq 1$ is:

$$S_n = a_1 \left(\frac{1 - r^n}{1 - r} \right)$$

(E4.) Consider the geometric series $1 + 5 + 25 + 125 + 625 + \dots$. Find the sum of the first 10 terms.

(P4.) Consider the geometric series $4 + 2 + 1 + \frac{1}{2} + \dots$. Find the sum of the first 10 terms.

(E5.) In 1990 the average monthly bill for cellular telephone service in the United States was \$80.90. From 1990 through 1997, the average monthly bill decreased by about 8.6% per year. *Source: Statistical Abstract of the United States*. Write a rule for the average monthly cellular telephone bill a_n (in dollars) in terms of the year. Let $n = 1$ represent 1990.

(P5.) You buy a new car for \$25,000. The value of the car decreases by 16% each year? Write a rule for the average yearly value of the car a_n (in dollars) in terms of the year. Let $n =$ the current year.

(E6.) Find the sum of the series

$$\sum_{i=1}^{10} 2(2)^{i-1}$$

(P6.) Find the sum of the series

$$\sum_{i=1}^{12} 3(4)^{i-1}$$

Vocabulary

_____ : Facts, observations and information that come from investigations.

- _____ (aka – measurement data) – data that has arithmetic calculations (i.e. test scores, weight)
- _____ (aka – categorical data) – data that is organized into different groups (i.e. males/females, favorite class)

_____ : a description of a group based the data of an _____ population

_____ : the total set of observations that can be made

Example: 10% of US senators voted for a particular measure. (there are only 100 senators)

_____ : a description of a group based on the data of a _____ population

_____ : a small portion of the entire population

_____ : the number of people in a population surveyed

_____ : a sample in which each individual is chosen entirely by chance and each member of the population has an equal chance of being included

_____ : a sample that is not a _____ sample. It is a sample collected in such a way that some members of the intended population are less likely to be included than others

Example: 30% of dog owners scoop poop after their dog.

Statistic or Parameter?

(E1.) 45% of Jacksonville, FL residents report that they have been to at least one Jaguars game.

(E2.) 40% of 1,211 students at a particular elementary school got below a 3 on a standardized test.

(E3.) 33% of 120 workers at a particular bike factory were paid less than \$20,000 per year.

(E4.) 60% of US residents agree with the latest health care proposal.

A statistic is an _____ based on a _____ population. This inference comes with a level of uncertainty. Statisticians account for this uncertainty using a variety of concepts including _____, _____, *and* _____ *of* _____.

_____ : a projected range in which the actual results of a statistic should fall. The range is derived using a _____ and a _____ *of* _____. The **confidence level** is somewhat arbitrary and most commonly represented as 90%, 95% and 99%. As the **margin of error** increases the **confidence level** increases. The **margin of error** is expressed as +/- percent.

You will learn to calculate a confidence interval in AP Stat and/or Stat

(E5.) A sample of students was selected to answer a survey question changes to the athletic program. The results showed that 72% of the students agreed with the change with a margin of error of 4%. Find the upper and lower bounds of the confidence interval.

(E6.) A poll found a confidence interval of 20% to 26% of county residences approve of a proposed bill. Identify the center of the interval. Identify the margin of error of the interval.

(E7.) 20% of students surveyed reported that they applied to Penn State. The margin of error is 2%. Find the upper and lower bounds of the confidence interval.

Data can sometimes be modeled by a function. Drawing a scatter plot of the data can help you recognize the type of function that best models the data. You can then use one of the regression features on a graphing calculator to find and graph an equation of the best-fitting model.

(E1).

Use a graphing calculator to draw a scatter plot of the data. Then tell whether a *linear*, *quadratic*, or *exponential* function would best model the data.

a. Average total cost y of a year of college, where $x = 3$ represents 1993:

x	3	4	5	6	7	8	9	10
y	7931	8306	8800	9206	9588	10,076	10,444	10,876

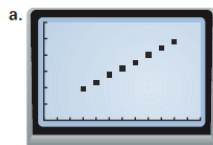
b. Number of customers y in a restaurant each hour, where $x = 3$ represents 3 P.M.:

x	3	4	5	6	7	8	9	10
y	15	30	40	50	45	42	31	18

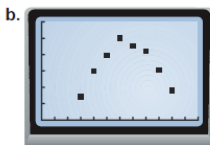
c. Population y of bacteria in a petri dish after x hours:

x	1	2	3	4	5	6
y	3	15	35	80	300	740

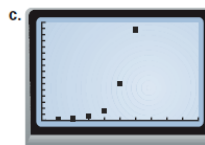
SOLUTION



The points lie nearly in a straight line. This suggests a linear model.

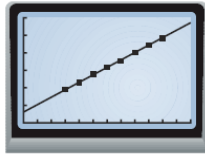
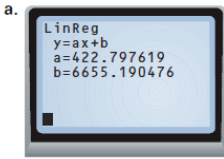


The points show a parabolic trend. This suggests a quadratic model.

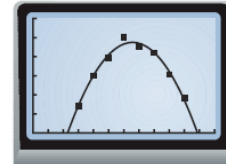
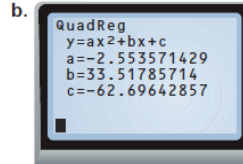


The points lie in a curve that seems to have an asymptote. This suggests an exponential model.

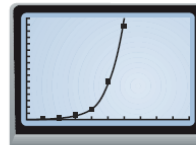
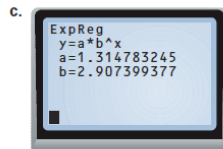
(E2.) Find and graph an equation of the best-fitting model for each data set in Example 1.



The linear regression equation is $y = 423x + 6660$.



The quadratic regression equation is $y = -2.55x^2 + 33.5x - 62.7$.



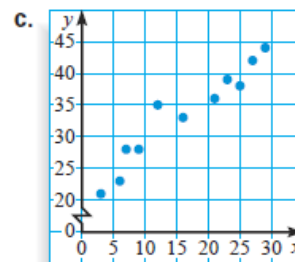
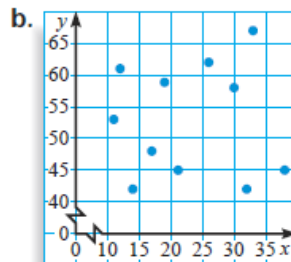
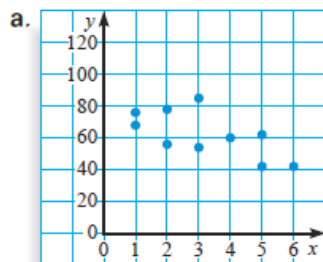
The exponential regression equation is $y = 1.31 \cdot 2.91^x$.

(E3.) Use the models in parts (a) and (b) of Example 2.

- Predict the average total cost of a year of college in 2007.
- Predict the number of customers in the restaurant at 6:30 P.M.

The **correlation coefficient** r for a set of paired data is a measure of how well a linear function models the data. If all of the graphed data pairs lie exactly on a line with a positive slope, the correlation coefficient is 1. If all of the graphed data pairs lie exactly on a line with a negative slope, the correlation coefficient is -1 . If the graphed data pairs tend not to lie on any line, the correlation coefficient is close to 0.

(E4.) Estimate the correlation coefficient for the data.



General Knowledge – basic terminology and notation used while calculating summary statistics

Data Points (n) – the number of points in a set found by counting the number of points in the set

Lower Quartile (Q_1) – the 25th percentile and is found by locating the median of the lower half of the data

Upper Quartile (Q_3) – the 75th percentile and is found by locating the median of the upper half of the data

Minimum Value (minX) – the lowest value in a set of data

Maximum Value (maxX) – the highest value in a set of data

Deviation (not generated by the calculator) – the difference between each data point and the mean

Outliers (not generated by the calculator) – an extreme value that is more than 1.5 times the interquartile range (IQR) beyond Q_1 or Q_3 . All potential outliers are all values outside of the “fences”:

$$Q_1 - 1.5IQR \quad \text{and} \quad Q_3 + 1.5IQR$$

5-Number Summary: This summary includes the minimum, first quartile, median, third quartile and maximum

Measures of Central Tendency - A number meant to convey the idea of “centralness” for a data set. The most commonly used measures of central tendency are the mean, median and mode.

Mean (\bar{x}) – the arithmetic average of the data set. It is calculated by summing all of the data points and then dividing by the number of data points.

Median (Med) – the 50th percentile. It is found by locating the center data point. If there are an even number of data points. There will then be two center data points. In this case, find the median by averaging the two center data points.

Mode (not generated by the calculator) – the data point that occurs the most frequently.

Measures of Variability - A number that is meant to convey the idea of spread for a set of data

Range (not generated by the calculator) - Maximum value minus the minimum value

Interquartile Range (not generated by calculator) - measures the spread of the middle 50 percent of an ordered data set. It is found by $Q_3 - Q_1$

Variance (not generated by the calculator) - the average* of the squares of the deviations of the data from their mean. It is also referred to as the square of the standard deviation and denoted s^2 .

$$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

Standard Deviation (Sx) - It is a measure of how spread out or varied the data is. It is found by taking the square root of the variance and denoted s

(E1.) Geometry Unit 1 Test Scores

{81, 77, 91, 90, 99, 92, 92, 84, 82, 27, 83, 99, 90, 79.5, 88.5, 76, 70, 99, 85, 56, 56, 50}

Mean:	Mode:	Median:	Minimum:	Maximum:
Quartile 1:	Quartile 3:	IQR:	Range:	Variance:
Standard Deviation:	1.5IQR	$Q_1 - 1.5IQR$:	$Q_3 + 1.5IQR$:	Outliers:

(P1.) Salaries

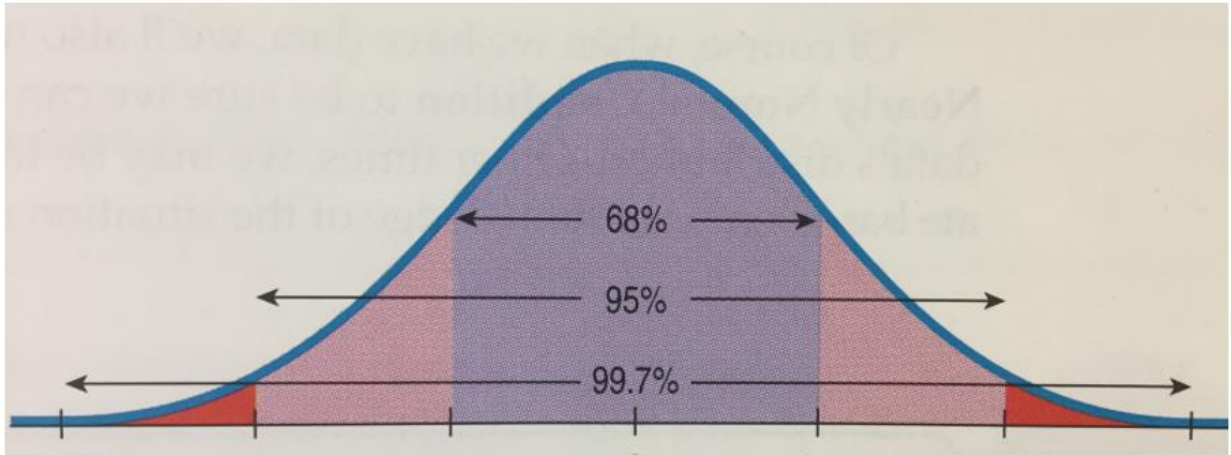
{73000, 75000, 75000, 75000, 80000, 80000, 82000, 82000, 84000, 85000, 85000, 89000, 90000, 91000, 91000, 92000, 92000, 94000, 94000, 100000, 105000, 200000, 300000}

Mean:	Mode:	Median:	Minimum:	Maximum:
Quartile 1:	Quartile 3:	IQR:	Range:	Variance:
Standard Deviation:	1.5IQR	$Q_1 - 1.5IQR$:	$Q_3 + 1.5IQR$:	Outliers:

Normal Distribution - a frequency distribution that results in a normal curve (aka "bell curve"). The important values for sketching and using a Normal Curve are the Mean and Standard Deviation

The 68-95-99.7 Rule

within 1 st.dev. of \bar{x} : 68%
within 2 st.dev. of \bar{x} : 95%
within 3 st.dev. of \bar{x} : 99.7%



(E2.) Given the below set of quiz scores, find the mean and standard deviation, and explain what it means. You can make a simple normal distribution graph.

{90, 88, 87, 95, 84, 86, 81, 99, 56, 86, 86, 88, 77, 94, 79, 74, 92, 88, 86, 95, 84, 92, 81, 85, 75, 99}

Mean: _____
Standard Deviation: _____

(P2.) Given the below set of test scores, find the mean and standard deviation, and explain what it means. Make a normal distribution graph.

{54.5, 70, 77, 36, 84, 51, 93, 90, 53.5, 89.5, 87.5, 83, 64.5, 89.5, 44, 81, 82.5, 60.5, 77, 97, 82, 48, 85, 60.5, 103, 71.5, 50, 82.5}

Mean: _____
Standard Deviation: _____

Applications:

(P3.) A student scored an 83%, 85% and 90% on three tests. What would they have to score on a fourth test, to have an overall test average of 88%?

(P4.) Suppose there were two more tests to be completed.

a) What would they have to score on the fourth and fifth test to have an overall average of 88%?

b) Find two possible test scores for the fourth and fifth test that would allow the overall test average to be an 88%.

(P5.) Find the average of all consecutive integers from 1 – 100.

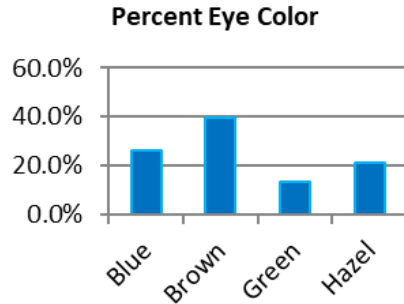
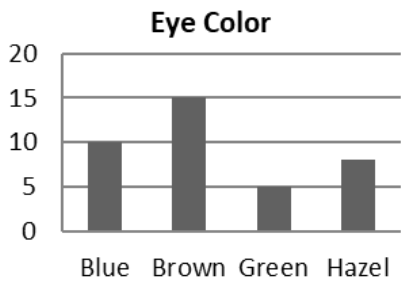
QUALITATIVE DATA DISPLAYS

_____ Data Displays – displays data that is organized by:

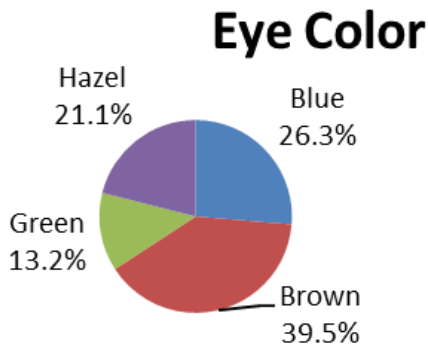
_____ – the _____ of the number of _____ that an event _____
OR

_____ – the _____ of the number of times an event _____
 _____ to the _____ of _____ on which it _____ occur. Relative frequency can
 be expressed as a _____, _____ or _____

Bar Graph – Vertical or horizontal bars that represent the _____ or _____ for each value of the variable.



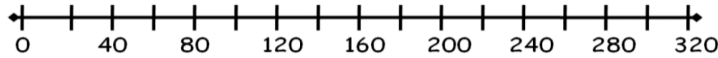
Pie Chart – A _____ represents the whole and each _____ of the variable is a _____ of the circle.
 Can only be used when comparing parts to the whole



QUANTITATIVE DATA DISPLAYS

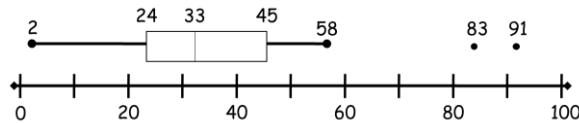
_____ and _____ Plots – a graphic way to display the _____, _____ and _____ of a data set on a number line to show the _____ of the data

(E1.) – Organize the following data into a Box and Whisker Plot. {4, 233, 15, 4, 197, 1, 231, 285, 278, 39}



Minimum:	Q_1 :	Median:	Q_3 :	Maximum:
IQR:	$1.5IQR$	$Q_1 - 1.5IQR$:	$Q_3 + 1.5IQR$:	Outliers:

(E2.) – The graph represents the amount of money surveyed people spent in a month at Starbucks.



Minimum:	Q_1 :	Median:	Q_3 :	Maximum:
IQR:	$1.5IQR$	$Q_1 - 1.5IQR$:	$Q_3 + 1.5IQR$:	Outliers:

What percent of the data is above \$45? _____

What would be your best prediction for the mean? Why? _____

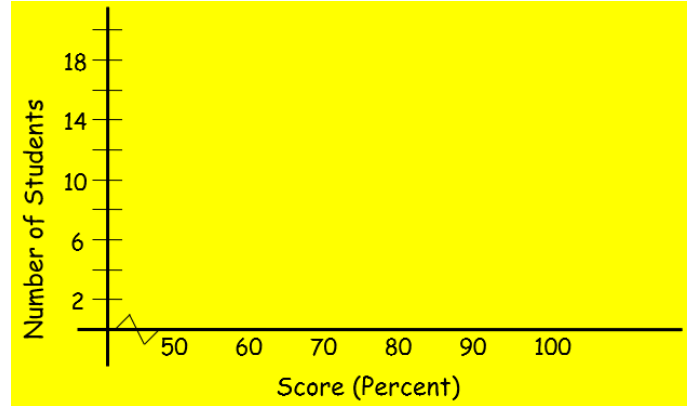
_____ and _____ Plot – a table in which data values are divided into either a “leaf” or a “stem”

(E3.) – Create a Stem and Leaf plot {81, 72, 63, 65, 80, 54, 92, 88, 72, 71, 66, 80, 83, 59, 50, 94}

Histogram – a _____ frequency distribution that is drawn over an _____ using rectangles. Each rectangle represents a _____ (subgroup of data) and has an area that is _____ to the _____ of a variable and a width that is equal to the _____ (the range of each class). Each class interval has an _____ and _____ value known as _____. If one data point falls ON a limit, it is counted in the _____ class.

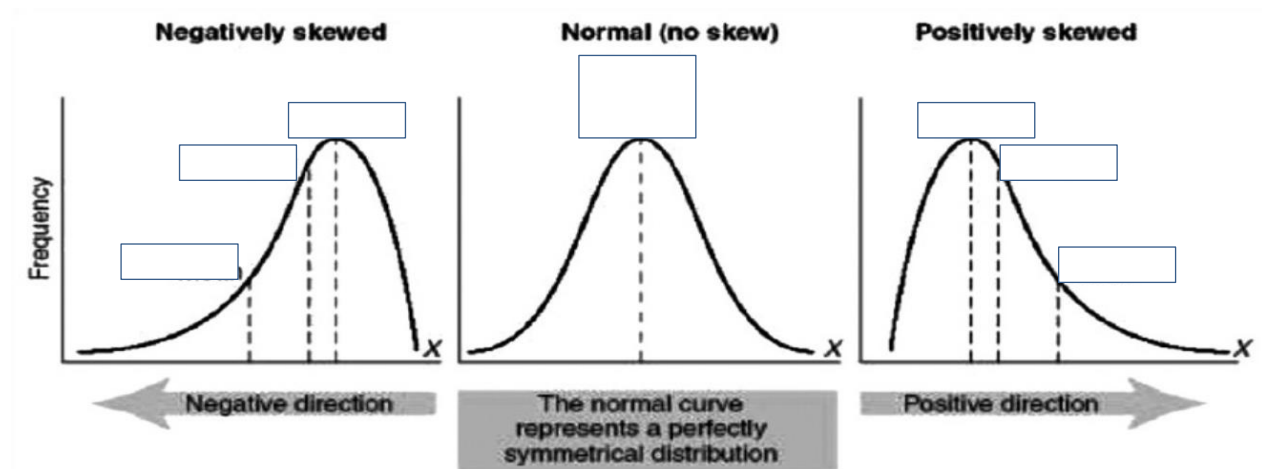
(E4.) Draw a Histogram to illustrate quiz scores.
 56, 74, 75, 77, 79, 81, 81, 84, 84, 85, 86, 86, 86,
 86, 87, 88, 88, 88, 90, 92, 92, 94, 95, 95, 99, 99

Score (%)	Number of Students
50 - 60	
60 - 70	
70 - 80	
80 - 90	
90 - 100	



SPREAD OF DATA

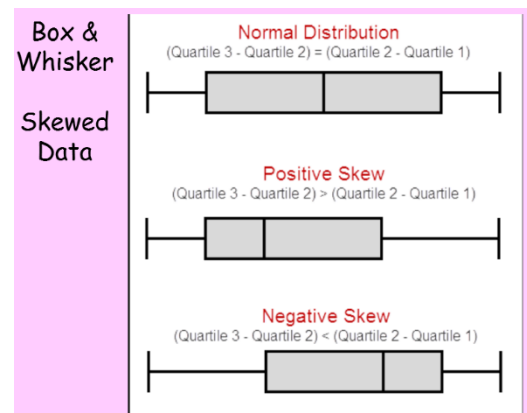
There are many ways to measure the spread of data. Statisticians often accept Interquartile range (IQR) as the best measure



NOTE: COMMON MISCONCEPTION – getting Left Skewed and Right Skewed flipped!
 TIP: **The “tail” points towards the direction of the skew.** Or “on what side is the mean higher?”

Stem & Leaf - Skewed Data		
SYMMETRIC DATA	RIGHT SKEWED	LEFT SKEWED
3 7	3 58	3 0
4 36	4 013345569	4 5
5 017	5 2446	5
6 1120	6 17	6 2
7 356789	7	7 17
8 0158	8 2	8 2445
9 43	9 5	9 012235569
10 0	10 0	10 000

TIP: Turn graphs sideways!



(E5.) Two teachers in the sociology department, Michelle and Bonnie, asked their students to record how many different websites they visited each day. The box and whisker plots below show the results.



- a) Which class has a higher median? _____
 b) Which class has a wider spread? _____
 c) Overall, which class is visiting more websites on a daily basis? Explain and/or support your answer.

d) Describe the skewness of each class. _____

(E6.) The below Stem-and-Leaf Plot represents the ages of tenured faculty members at Penn State University.

- a) What is the median age of tenured faculty members?

3	2 7 9
4	0 2 3 5 6 7 7 8 9
5	0 1 1 2 3 3 3 4 4 4 6 6 9 9
6	0 0 1 1 2 2 3 3 3 5 8

- b) Describe the skewness of the graph.

When dealing with the occurrence of more than one event or activity, it is important to be able to quickly determine how many possible outcomes exist.



For example, if ice cream sundaes come in 5 flavors with 4 possible toppings, how many different sundaes can be made with one flavor of ice cream and one topping?

Rather than list the entire sample space with all possible combinations of ice cream and toppings, we may simply multiply: $5 \cdot 4 = 20$ possible sundaes. This simple multiplication process is known as the **Counting Principle**.

The Fundamental Counting Principle: If there are **a** ways for one activity to occur, and **b** ways for a second activity to occur, then there are **$a \cdot b$** ways for both to occur.

The fundamental counting principle can be extended to three or more events. For example, if three events can occur in m , n , and p ways, then the number of ways that all three events can occur is $m \cdot n \cdot p$.

For instance, if three events can occur in 2, 5, and 7 ways, then all three events can occur in $2 \cdot 5 \cdot 7 = 70$ ways.

(E1.) Police use photographs of various facial features to help witnesses identify suspects. One basic identification kit contains 195 hairlines, 99 eyes and eyebrows, 89 noses, 105 mouths, and 74 chins and cheeks.

(a) The developer of the identification kit claims that it can produce billions of different faces. Is this claim correct?

(b) A witness can clearly remember the hairline and the eyes and eyebrows of a suspect. How many different faces can be produced with this information?

(P1.) In a high school there are 273 freshmen, 291 sophomores, 252 juniors and 237 seniors. In how many different ways can a committee of 1 freshman, 1 sophomore, 1 junior and 1 senior be chosen?

- (E2.) The standard configuration for a New York license plate is 3 digits followed by 3 letters.
- (a) How many different license plates are possible if digits and letters can be repeated?
- (b) How many different license plates are possible if digits and letters cannot be repeated?
- (P2.) A multiple choice test has 10 questions with 4 answer choices for each question. In how many different ways could you complete the test?

A **permutation** is an arrangement of objects in specific order.

The order of the arrangement is important!!



Consider, four students walking toward their school entrance. How many different ways could they arrange themselves in this side-by-side pattern?

1,2,3,4	2,1,3,4	3,2,1,4	4,2,3,1
1,2,4,3	2,1,4,3	3,2,4,1	4,2,1,3
1,3,2,4	2,3,1,4	3,1,2,4	4,3,2,1
1,3,4,2	2,3,4,1	3,1,4,2	4,3,1,2
1,4,2,3	2,4,1,3	3,4,2,1	4,1,2,3
1,4,3,2	2,4,3,1	3,4,1,2	4,1,3,2

The number of different arrangements is 24 or $4! = 4 \cdot 3 \cdot 2 \cdot 1$. There are 24 different arrangements, or permutations, of the four students walking side-by-side.

The notation for a permutation:

$n P r$

n is the total number of objects

The formula for a permutation:

$${}_n P_r = \underbrace{n(n-1)(n-2)\dots}_{r \text{ factors}}$$

OR

The formula for a permutation:

$${}_n P_r = \frac{n!}{(n-r)!}$$

(Remember that $0! = 1$.)

(Note: if $n = r$, as it did with the students walking side-by-side, then ${}_n P_r = n!$ for either formula.)

(E3.) Twelve skiers are competing in the final round of the Olympic freestyle skiing aerial competition.

(a) In how many different ways can the skiers finish the competition? (Assume there are no ties.)

(b) In how many different ways can 3 of the skiers finish first, second and third to win the gold, silver and bronze medals?

(P3.) You have homework assignments from 5 different classes to complete this weekend.

(a) In how many different ways can you complete the assignments?

(b) In how many different ways can you choose 2 of the assignments to complete first and last?

(E4.) You are considering 10 different colleges. Before you decide to apply to the colleges, you want to visit some or all of them. In how many orders can you visit:

(a) 6 of the colleges?

(b) all 10 colleges?

(P4.) There are 12 books on the summer reading list. You want to read some or all of them. In how many different orders can you read:

(a) 4 of the books?

(b) all 12 of the books?

A combination is a selection of r objects from a group of n objects where the order is not important. For instance, in most card games the order in which your cards are dealt is NOT important.

$${}_n C_r = \binom{n}{r} = \frac{{}_n P_r}{r!} = \frac{n!}{r!(n-r)!}$$

For instance, the number of combinations of 2 objects taken from a group of 5 objects is ${}_5 C_2 = \frac{5!}{3!2!} = 10$

(E1.) A standard deck of 52 playing cards has 4 suits with 13 different cards in each suit. If the order in which the cards are dealt is not important, how many different 5 card hands are possible?

Standard 52-Card Deck

K ♠	K ♣	K ♦	K ♥
Q ♠	Q ♣	Q ♦	Q ♥
J ♠	J ♣	J ♦	J ♥
10 ♠	10 ♣	10 ♦	10 ♥
9 ♠	9 ♣	9 ♦	9 ♥
8 ♠	8 ♣	8 ♦	8 ♥
7 ♠	7 ♣	7 ♦	7 ♥
6 ♠	6 ♣	6 ♦	6 ♥
5 ♠	5 ♣	5 ♦	5 ♥
4 ♠	4 ♣	4 ♦	4 ♥
3 ♠	3 ♣	3 ♦	3 ♥
2 ♠	2 ♣	2 ♦	2 ♥
A ♠	A ♣	A ♦	A ♥

(P1.) Using the standard deck mentioned above, if the order is not important, how many different 7 card hands are possible?

IMPORTANT NOTE:

- (1) When finding the number of ways both an event A **AND** an event B can occur, you need to multiply.
- (2) When finding the number of ways that an event A **OR** an event B can occur, you add instead.

(E2.) A restaurant serves omelets that can be ordered with any of the ingredients shown. Suppose you want *exactly* 2 vegetarian ingredients and 1 meat ingredient in your omelet. How many different types of omelets can you order?

Omelets \$3.00 (plus \$.50 for each ingredient)	
<u>Vegetarian</u>	<u>Meat</u>
green pepper	ham
red pepper	bacon
onion	sausage
mushroom	steak
tomato	
cheese	

(P2.) You are taking a vacation. You can visit as many as 5 different cities and 7 different attractions. Suppose you want to visit exactly 3 different cities and 4 different attractions. How many different trips are possible?

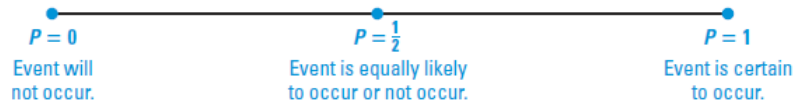
One of the stumbling blocks that students face when dealing with permutations and combinations is knowing whether the problem requires a combination or a permutation. Here are some examples of when each is applicable. Notice, in permutations, the order IS important and in combinations, the order is NOT important.

Permutation versus Combination	
1. Picking a team captain, pitcher, and shortstop from a group.	1. Picking three team members from a group.
2. Picking your favorite two colors, in order, from a color brochure.	2. Picking two colors from a color brochure.
3. Picking first, second and third place winners.	3. Picking three winners.

****Some Key Words For Permutations: Arrange, Order, Line up****

****Some Key Words For Combinations: Select, Group, Choose****

The probability of an event is a number between 0 and 1 that indicates the likelihood that an event will occur. An event that is certain to occur has a probability of 1. An event that cannot occur has a probability of 0. An event that is equally likely to occur or not occur has a probability of $\frac{1}{2}$.



Theoretical probability is based on knowing all of the equally likely outcomes of an experiment.

Probability of an Event

The probability of an event when all the outcomes are equally likely is:

$$P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}}$$

Sample Space is also another word used for the number of all possible outcomes.

A probability that is based on repeated trials of an experiment is called an experimental probability. Each trial in which the event occurs is a success.

Experimental Probability

The experimental probability of an event is:

$$P(\text{event}) = \frac{\text{Number of successes}}{\text{Number of trials}}$$

(E1.) You roll a six-sided die whose sides are numbered from 1 through 6.

(a) Find the probability of rolling a 4

(b) Find the probability of rolling an odd number

(c) Find the probability of rolling a number less than 7

(P1.) A spinner has 8 equal-size sectors numbered from 1 to 8.

(a) Find the probability of spinning a 6

(b) Find the probability of spinning a number greater than 5

(E2.) You have a CD that has 8 songs in your CD player. You set the player to play the songs at random. The player plays all 8 songs without repeating any song.

(a) What is the probability that the songs are played in the same order they are listed on the CD?

(b) You have 4 favorite songs on the CD. What is the probability that 2 of your favorite songs are played first, in any order?

(P2.) There are 9 students on the math team. You draw their names one by one to determine the order in which they answer questions at a math meet. What is the probability that 3 of the 5 seniors on the team will be chosen last, in any order?

Odds When all outcomes are equally likely, the ratio of the number of favorable outcomes to the number of unfavorable outcomes is called the **odds in favor** of an event. The ratio of the number of unfavorable outcomes to the number of favorable outcomes is called the **odds against** an event.

$$\text{Odds in favor} = \frac{\text{Number of favorable outcomes}}{\text{Number of unfavorable outcomes}}$$

$$\text{Odds against} = \frac{\text{Number of unfavorable outcomes}}{\text{Number of favorable outcomes}}$$

(E3.) You randomly choose an integer from 0 through 9. What are the odds that the integer is 4 or more?

(P3.) You randomly choose a letter from the word SUMMER. What are the odds that the letter is a vowel?

(E4.) The probability that a randomly chosen household has a cat is 0.27. *Source: American Veterinary Medical Association.* What are the odds

(a) that a household has a cat?

(b) that a household does NOT have a cat?

(P4.) The probability that a randomly chosen 4 digit security code contains at least one zero is 0.34. What are the odds that a 4 digit security code contains at least one zero?

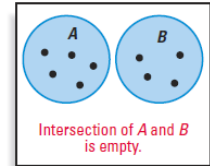
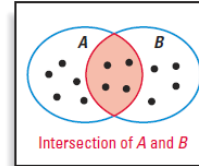
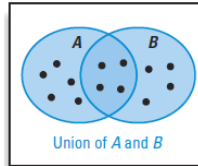
Probabilities of Unions and Intersections

VOCABULARY

Compound event The union or intersection of two events

Mutually exclusive events Events A and B are mutually exclusive if the intersection of A and B is empty.

Complement The complement of event A, denoted A', consists of all outcomes that are not in A.



PROBABILITY OF COMPOUND EVENTS

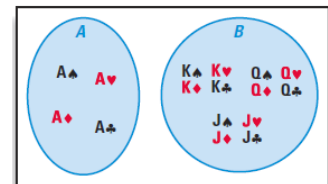
If A and B are two events, then the probability of A or B is:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

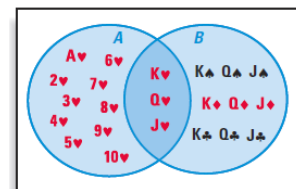
If A and B are mutually exclusive, then the probability of A or B is:

$$P(A \text{ or } B) = P(A) + P(B)$$

(E1.) A card is randomly selected from a standard deck of 52 cards. What is the probability that it is an ace or a face card? (MEE)



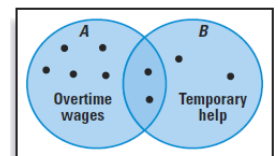
(P1.) One six sided die is rolled. What is the probability of rolling a multiple of 3 or 5? (MEE)



(E2.) A card is randomly selected from a standard deck of 52 cards. What is the probability that the card is a heart or a face card?(NMEE)


(P2.) One six sided die is rolled. What is the probability of rolling a multiple of 3 or a multiple of 2? (NMEE)

(E3.) Last year a company paid overtime wages or hired temporary help during 9 months. Overtime wages were paid during 7 months and temporary help was hired during 4 months. At the end of the year, an auditor examines the accounting records and randomly selects one month to check the company's payroll. What is the probability that the auditor will select a month in which the company paid overtime wages and hired temporary help?



(P3.) In a poll of high school juniors, 6 out of 15 students took a French class and 11 out of 15 took a math class. Fourteen out of 15 students took French or math. What is the probability that a student took both French and math?

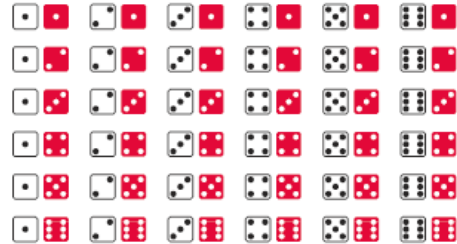
The probability of the complement of an event is one minus the probability of the event.



Complement:
 $P(A') = 1 - P(A)$

(E4.) When two six sided dice are tossed, there are 36 possible outcomes as shown. Find the probability of the given event.

(a) The sum is NOT 8



(b) The sum is greater than or equal to 4

(P4.) A card is randomly selected from a standard deck of 52 cards. Find the probability of the given event.

(a) The card is NOT a king

(b) The card is NOT an ace or a jack

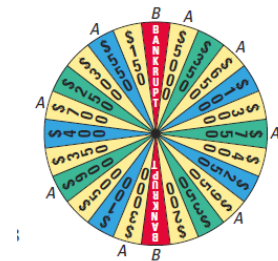
Two events are said to be **independent** if the result of the second event is not affected by the result of the first event.

If A and B are independent events, the probability of **both** events occurring is the product of the probabilities of the individual events



If A and B are **independent** events,
 $P(A \text{ and } B) = P(A) \cdot P(B)$.

(E1.) You are playing a game that involves spinning the money wheel shown. During your turn you get to spin the wheel twice. What is the probability that you get more than \$500 on your first spin and then go bankrupt on your second spin?



(P1.) A game machine claims that 1 in every 15 people win. What is the probability that you win twice in a row?

(E2.) During the 1997 baseball season, the Florida marlins won 5 out of 7 home games and 3 out of 7 away games against the San Francisco Giants. During the 1997 National League Division Series with the Giants, the Marlins played the first two games at home and the third game away. The Marlins won all three games. Estimate the probability of this happening.

(P2.) In a survey 9 out 11 men and 4 out of 7 women said they were satisfied with a product. If the next three customers are 2 women and a man, what is the probability that they will all be satisfied?

If the result of one event **IS** affected by the result of another event, the events are said to be **dependent**.

If A and B are dependent events, the probability of both events occurring is the product of the probability of the first event and the probability of the second event once the first event has occurred.

If A and B are **dependent** events,
and A occurs first,
 $P(A \text{ and } B) = P(A) \cdot P(B, \text{once } A \text{ has occurred})$
... and is written as ...
 $P(A \text{ and } B) = P(A) \cdot P(B|A)$



(E3.) You randomly select two cards from a standard 52-card deck. What is the probability that the first card is not a face card (a king, queen or jack) and the second card is a face card if
(a) you replace the first card before selecting the second card

(b) you do NOT replace the first card

(P3.) You randomly select two cards from a standard 52-card deck. Find the probability that the first card is a diamond and the second card is red if
(a) you replace the first card before selecting the second card

(b) you do NOT replace the first card

(E4.) You and two friends go to a restaurant to order a sandwich. The menu has 10 types of sandwiches and each of you is equally likely to order any type. What is the probability that each of you orders a different type?

(P4.) Three children have a choice of 12 summer camps that they can attend. If they each randomly choose which camp to attend, what is the probability that they attended all different camps?

A *two-way table* is a useful way to organize data that can be categorized by two variables.

		Political Preference			total
		Liberal	Moderate	Conservative	
Gender	Female	35		6	77
	Male	50	44		115
	total		80	27	

E1.) Find the missing pieces of the table.

E2.) What percent of people are male given that they are conservative?

E3.) What percent of females are liberal?

E4.) If we know someone is moderate, what is the chance (%) they are male?

E5.) What percent of people that are conservative are female?

E6.) What percent of people are male and liberal?

E7.) What percent of people are female and conservative?

E8.) A male student is surveyed randomly, what is the probability that the student is a conservative?

Warm-ups

Use the provided spaces to complete any warm-up problem or activity

Date:	Date:
Date:	Date:
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Warm-ups

Use the provided spaces to complete any warm-up problem or activity

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